**Assignment - Week 1 Day 4**

**Question 1.**

An array of size 100000000000 is filled An array of size 100000000000 is filled with four different letters A, B, C and D. Assume that all four letters are equally likely to appear in the array S. However, there is no guarantee that all four letters are in the array.

(a) What is the average number of array locations to inspect to find the first D? Give your answer using a formula or result mentioned in the class note. Please give the Slide number as a reference.

Solution:

As all four letters(A, B, C and D) are equally likely to appear in the array S.

Probability of each letter is 1/4

P(A), P(B), P(C), P(D) =1/4

Average number of array locations to inspect to find the first D =1/p =1/(1/4)=4

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(b) What is the average number of array locations to inspect to find 10 D’s? Give your answer using a formula or result mentioned in the class note. Please give the Slide number as a reference.

Solution:

Expected number (average number) of trials for k successes = 𝒌/p

So, average number of array locations to inspect to find 10 D’s =10/(1/4)=40

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(c) What is the “average time complexity” to find k D’s in an array?

Solution:

Expected number (average number) of trials for k successes = 𝒌/p

So, average number of trials to inspect to find k D’s =k/(1/4)=4k

The average number of trials to inspect to find k D’s is 4k. The time complexity is O(k).

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**Question 2.**

Prove: 1 + 1/2 + 1/3 + ...+ 1/n = O(log n).

**Hint:**

Let n = **7**

1+1/2+1/3+1/4+1/5+1/6+1/7<=1+1/2+1/2+1/4+1/4+1/4+1/4 =3=log(**7**+1)

Solution:

Let n=15

1+1/2+1/3+1/4+1/5+1/6+1/7+1/8+1/9+1/10+1/11+1/12+1/13+1/14+1/15 <= 1+1/2+1/2+1/4+1/4+1/4+1/4+1/8+1/8+1/8+1/8+1/8+1/8+1/8+1/8 =4=log(15+1)

Therefore for any n,

Therefore, 1 + 1/2 + 1/3 + ...+ 1/n = log(n+1) = O(log n)

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**Question 3.**

Find the sum: 1/2+2/4+3/8+4/16+5/32+...

**Hint:**

S = 1/2 + 2/4 + 3/8 + 4/16 + 5/32 + ...

S/2 = 1/4 +2/8+3/16+4/32+...

S–S/2 =1/2+1/4+1/8+1/16+1/32+...

Solution:

S = 1/2 + 2/4 + 3/8 + 4/16 + 5/32 + …

S/2 = 1/4 +2/8+3/16+4/32+...

S–S/2 =1/2+1/4+1/8+1/16+1/32+... It's a geometric series. So, S=a/1-r

S-S/2 = (½)/(1-½)=1

S(1-½)=1

S(½)=1

S=2